

CIAO 2005

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New Directions in the Ω MEGA Project

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joint work with: The OMEGA Group

Issues of this Talk



- History and Aims of the OMEGA Project
 - A New Logic Layer: CORE
 - A New Interface to the Logic Layer: TASKS
 - An Improved Proof Datastructure: PDS
 - Adding Management of Change: MAYA
 - Adaptation of Proof Planning, Agent-based Reasoning, User-Interaction
 - **A New User Interface: *TEX_{MACS}***
 - Bridging between (multimodal) NL Proofs and Logic-level Proofs

History and Aims of the OMEGA Project



- THE GOAL:

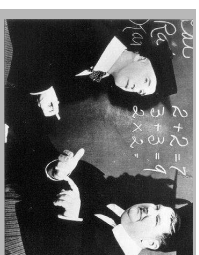


← teamwork →



- **integrated** mathematical assistant environment
- support for **typical tasks** of a mathematician

- THE PROBLEM:



- abstract-level representation →

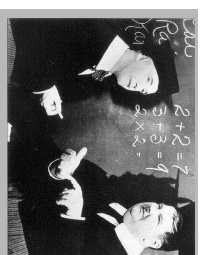
Miscommunication

← low-level representation -



- **resource** “mathematical knowledge” represented and employed differently
- majority in community: “human should adapt to computer”

- OUR APPROACH:

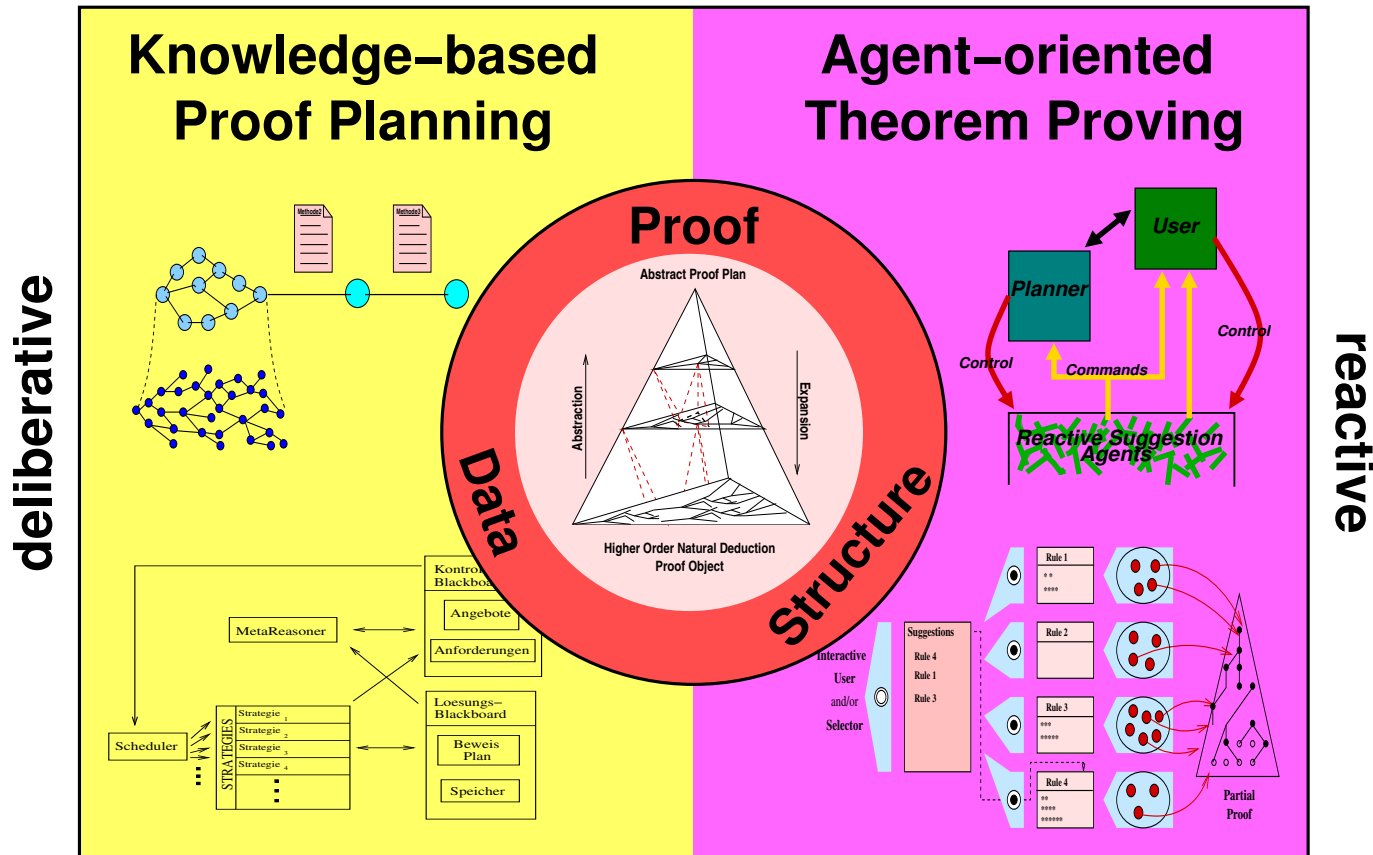


- abstract-level representation →

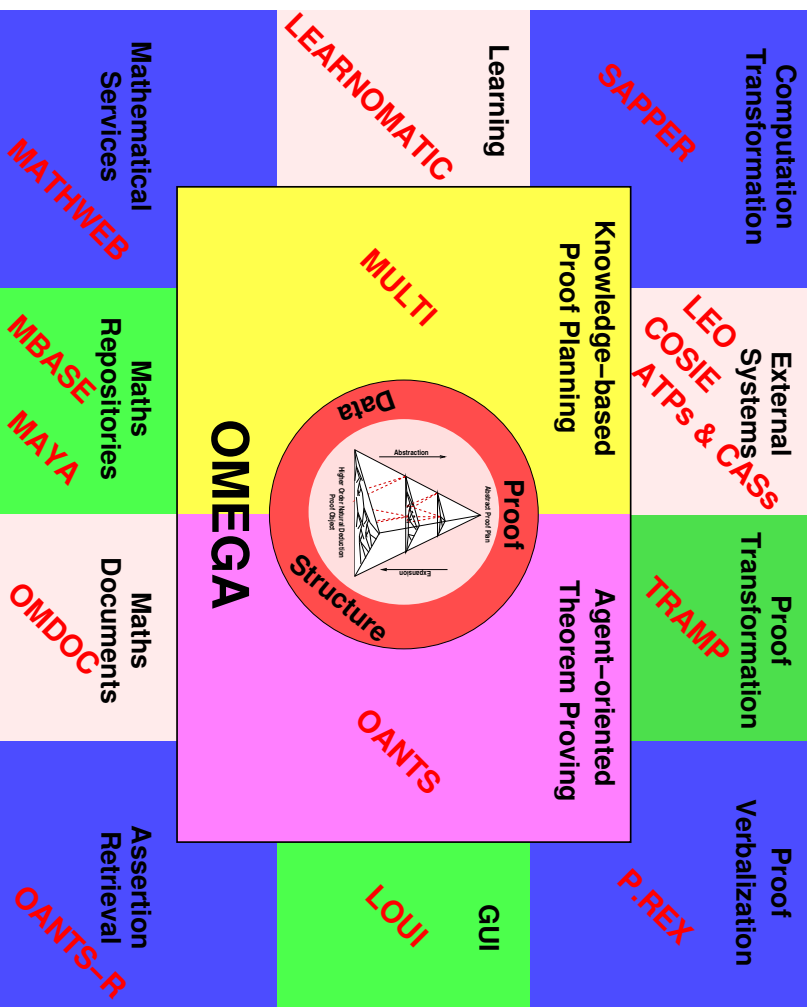
↑↑↑↑↑↑↑↑↑↑
← low-level representation -



History and Aims of the OMEGA Project



History and Aims of the OMEGA Project



COLLABORATIONS
(with joint publication)

- Yale University, USA
- CMU, Pittsburgh, USA
- University of Miami, USA
- The Univ. of Edinburgh, UK
- The Univ. of Birmingham, UK
- Imperial College London, UK
- Imperial College London, UK
- Eindhoven TU, Netherlands
- Cambridge University, UK
- International Univ. of Bremen, D
- University of Kaiserslautern, D
- TU Munich, D
- EU RTN CALCULEMUS

OMEGA Base Calculus: ND vs CORE



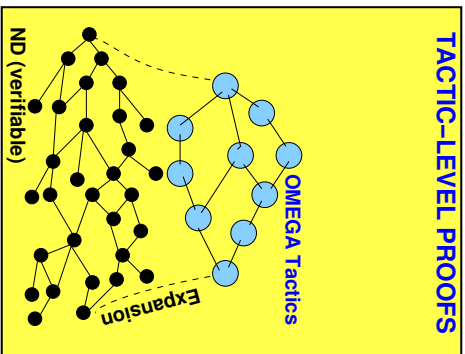
Theorem 1. $\sqrt{2}$ is irrational.

Proof. (by contradiction)

Assume $\sqrt{2}$ is rational, **that is**, there exist natural numbers m, n with no common divisor such that $\sqrt{2} = \frac{m}{n}$.

Then $n\sqrt{2} = m$, and **thus** $2n^2 = m^2$. **Hence** m^2 is even and, since odd numbers square to odds, **m is even**; say $m = 2k$.
...

□

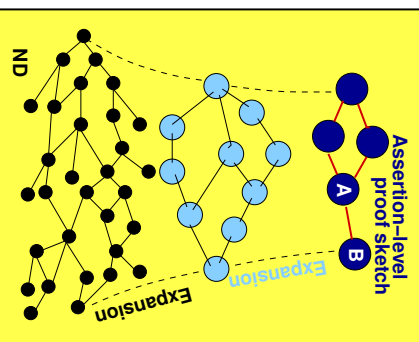


Result: **Tactic-based theorem provers** like PVS, ISABELLE, COQ, etc. (and OMEGA) can construct a proof ——— but **not at an adequate level**

A New Logic Layer: CORE



ISLAND PROOF PLANNING



SOLUTION A

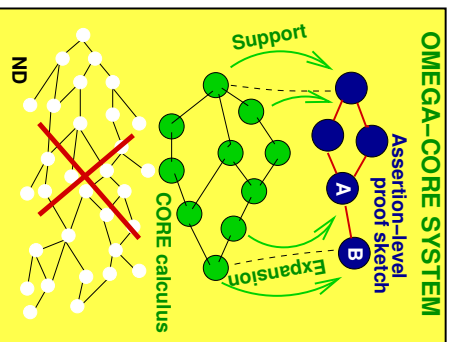
[35years-of-AUTOMATH]

- new explicit layer for proof sketches
 - easy to add to proof data structure
 - proof sketches may be unsound
 - verification by expansion to ND
- Problem:**
- expansion distance to base-layer

SOLUTION B

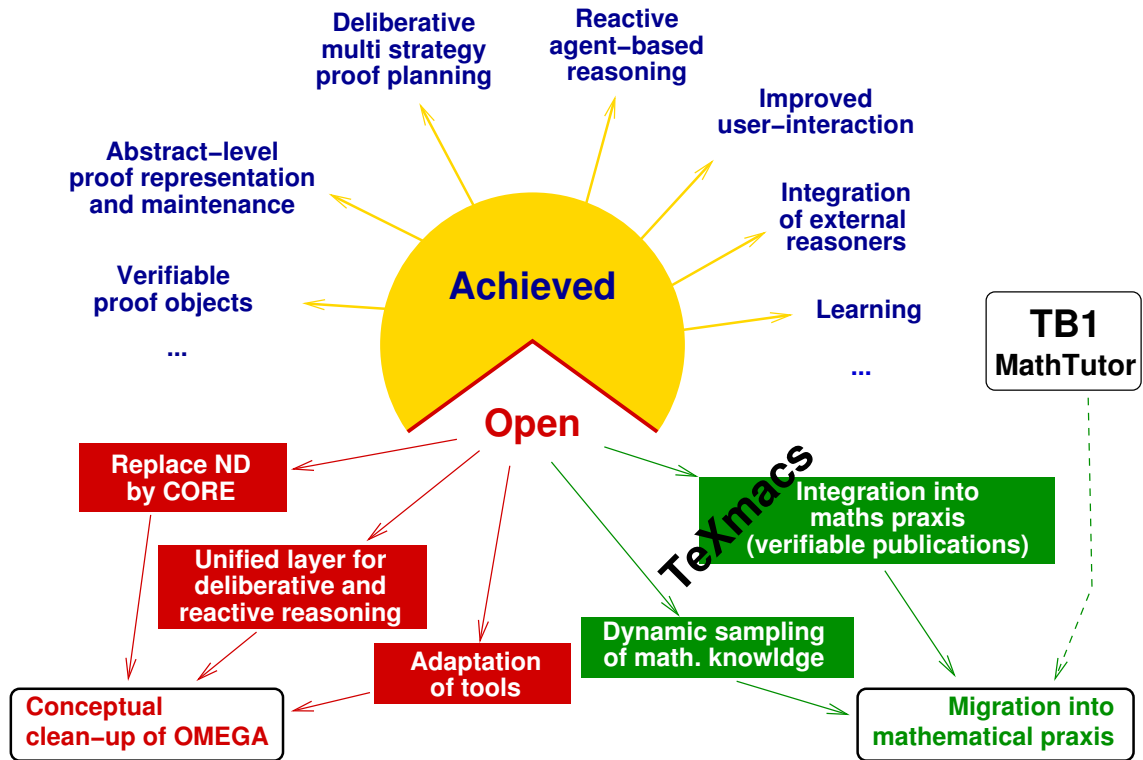
[PhD-Autexier-2003]

(realization started)



- CORE is more natural/abstract-level base calculus for proof assistants
- subsumes old ND base and (parts of) the old tactic layer
- provides more direct, constructive support for upper layer(s)

OMEGA Project: Open Tasks



A New Interface to the Logic Layer: TASKS



- Problem in the current OMEGA system:
 - interactive theorem proving
 - deliberative knowledge-based proof planning
 - reactive agent-based proof planning
- do **not employ a common interface** to the OMEGA base calculus
- OMEGA's current proof planner Multi based on “proof (plan) tasks”
 - maintained by Multi on an agenda
 - employed to structure a proof into subtasks
 - basically only a proof goal together with its supporting assertions

A New Interface to the Logic Layer: TASKS



Deliberative Knowledge-based Proof-Planning
Interactive Theorem Proving
Agent-based Proof Planning

Tasks

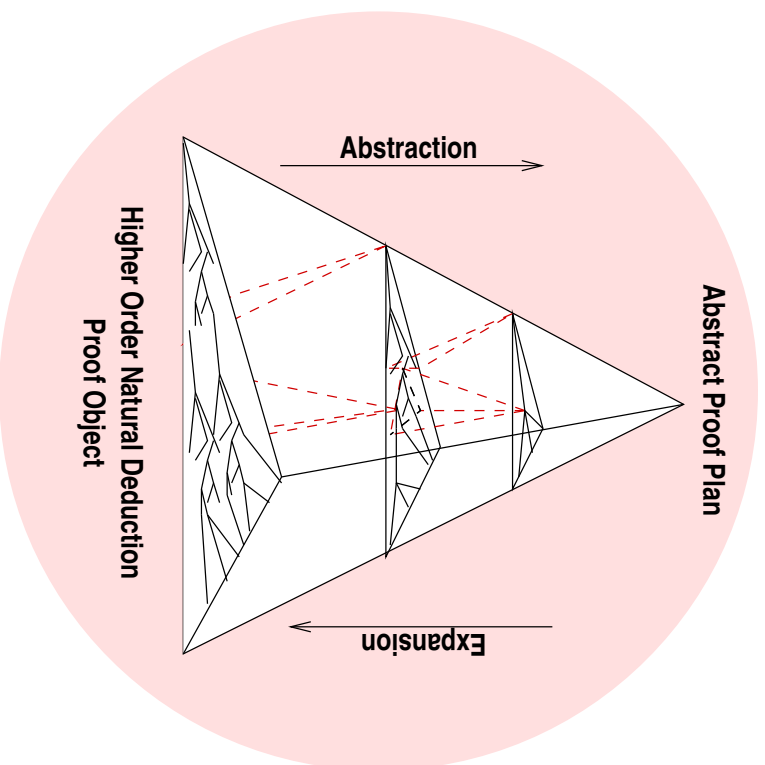
...
(rat-criterion) ...
(even) $\lambda x. \exists y. x = 2y$
(square-even) $\text{even}(x^2) \Rightarrow \text{even}(x)$
(goal) $\sqrt{2}$ is not rational

CORE calculus

Technically a task is a

- multi-conclusion sequent: $\text{ass}_1^-, \dots, \text{ass}_n^-, \text{goal}_1^+, \dots, \text{goal}_m^+$
(premise-conclusion-status defined via polarities which are maintained by CORE)
- with focus of attention: $\text{ass}_1^-, \dots, \text{ass}_n^-, \text{goal}_1^+, \dots, \text{goal}_m^+$

An Improved Proof Datastructure: PDS



Problems and Missing Features

- (loosely) dependent on ND calculus
- no proof forests (lemmata)
- no OR-branching

An Improved Proof Datastructure: PDS



- Separation of
 - PDS framework
 - from its instantiation by an inference system
- Instantiation of PDS framework for the new OMEGA with Proof Tasks
- Support for Proof Forests:
 - one proof tree for each lemma
 - taken from Quodlibet framework
- Addition of OR-branches: AND-OR proof trees

- Integration of OMEGA and MAYA
 - obtain support for management of change in theory exploration in mathematics
 - MAYA as tool between user and OMEGA
 - MAYA as tool for joint document development (multiple users)
- Extension of MAYA
 - support for other theory specific mathematical knowledge: proof tactics, proof methods, mathematical documents
- Important Sidegoal:
 - reunification/integration of the systems developed in the AGS via MAYA (OMEGA, INKA, VSE)

Adaptation of Proof Planning & Co



- Knowledge-based Proof-Planning (PP)
- Interactive Theorem Proving (ITP)
- Agent-based Proof Planning (APP)

-
-
-

Tasks

...
(rat-criterion) ...
(even) $\lambda x. \exists y. x = 2y$
(square-even) $\text{even}(x^2) \Rightarrow \text{even}(x)$
(goal) $\sqrt{2}$ is not rational

CORE calculus

What Benefits?

- Conceptual clean-up of OMEGA
 - proof planning with proof methods (and not wit
- Better intergration of PP, ITP, APP

Adaptation of Proof Planning & Co



OMEGA Interface: Why TEX_{MACS} ?



What is TEX_{MACS} ? :

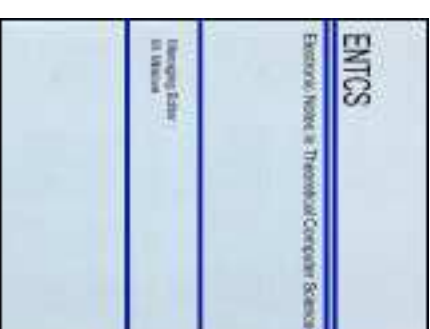
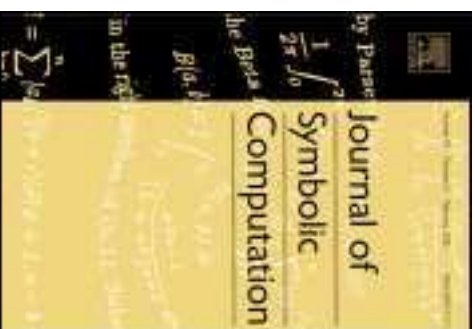
- symbiosis of LaTeX and Emacs; supports professional mathematical editing
- integrates many support tools: CASS (Maple ...), Gnuplot, ..., Scheme, Lisp, ...
- Springer official document format for maths journals (in test)
- **missing: integrated proof assistant (OMEGA)**

A small Demo: see [here!](#)

$$\sqrt[k]{\sqrt{4+8}}$$

This is an [example for an action](#)

OMEGA- T_{EX} M_{ACS} Interface: Our Vision



- Verifiable Scientific Documents (also Marking of Exams)
- Active Scientific Documents
- Overcome Need for Multiple Encodings of Mathematical Knowledge

A Short Demo



Human-Oriented Problem Representation	Formal Representation
<p>Chris invites Jörg, Claus-Peter, and Erica to his Party. He receives the following replies: Jörg: "Claus-Peter or Erica will come" Claus-Peter: "Either Jörg or Erica will come" Erica: "Either Jörg or Claus-Peter will come" Theorem: Erica will be at the Party.</p>	<p>Chris v. Erica (Joerg & ~Erica) v. (~Joerg & Erica) (Joerg & ~Chris) v. (~Joerg & Chris) "above axioms" = Erica</p>

Theorem: {Chris v. Erica, (Joerg & ~Erica) v. (~Joerg & Erica), (Joerg & ~Chris) v. (~Joerg & Chris)} |= Erica

Proof:

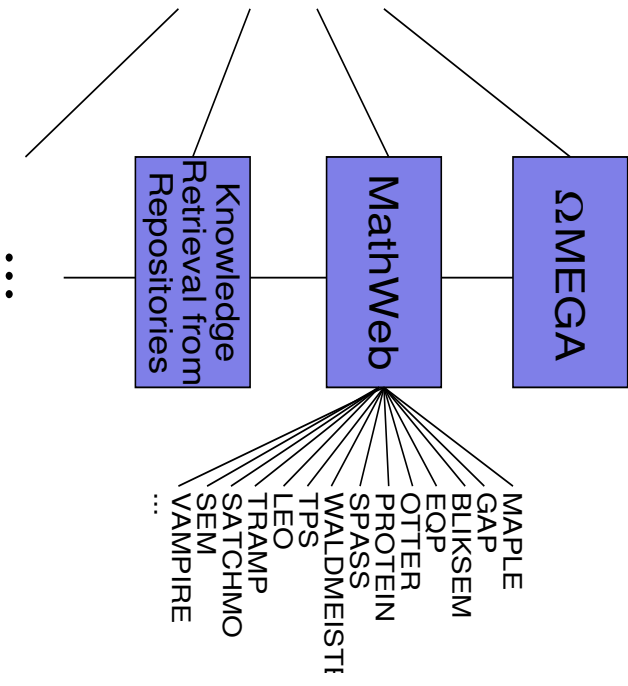
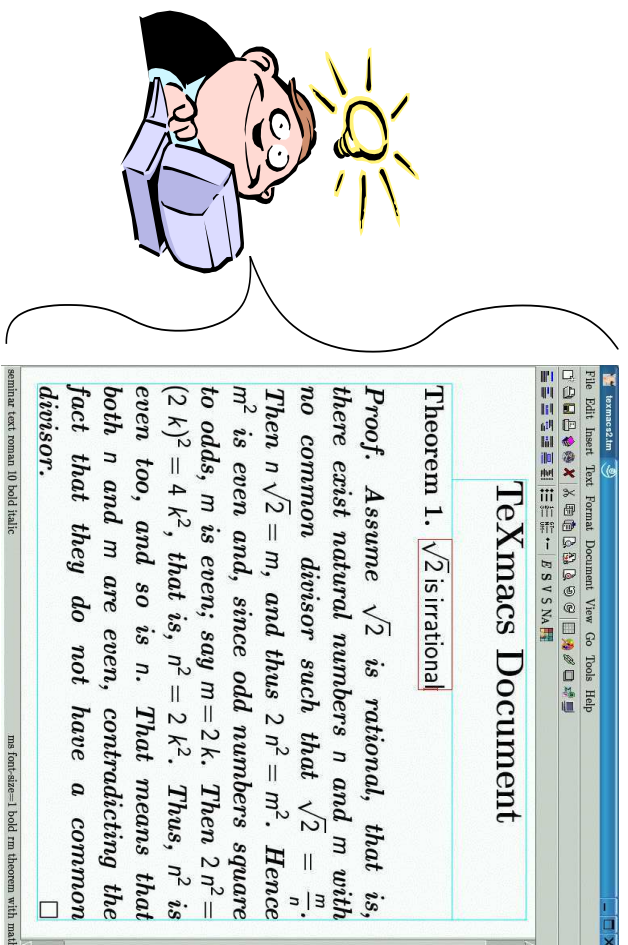
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"translatedBurf{{C}}{h}}{i}}{s}}{v}}{.}}{E}}{r}}{i}}{c}}{h}}{a}}{.}}{({}}{J}}{o}}{e}}{r}}{g}}{&}}{E}}{r}}{i}}{c}}{h}}{a}}{.}}{({}}{J}}{o}}{e}}{r}}{g}}{&}}{C}}{h}}{i}}{s}}{)}}{=}{{E}}{r}}{i}}{c}}{h}}{a}}"
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OMEGA- TEX_{MACS} Interface: Our Vision



- TEX_{MACS} (www.texmacs.org) as scientific wysiwyg Texteditor



- Bridging between NL proofs and Logic-level proofs: VeriMath & DIALOG

There is no conclusion yet