An Agent based Approach to (Mathematical) Reasoning

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joint work with

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Motivation *Cognitive Perspective*

To solve complex problems in mathematics or engineering

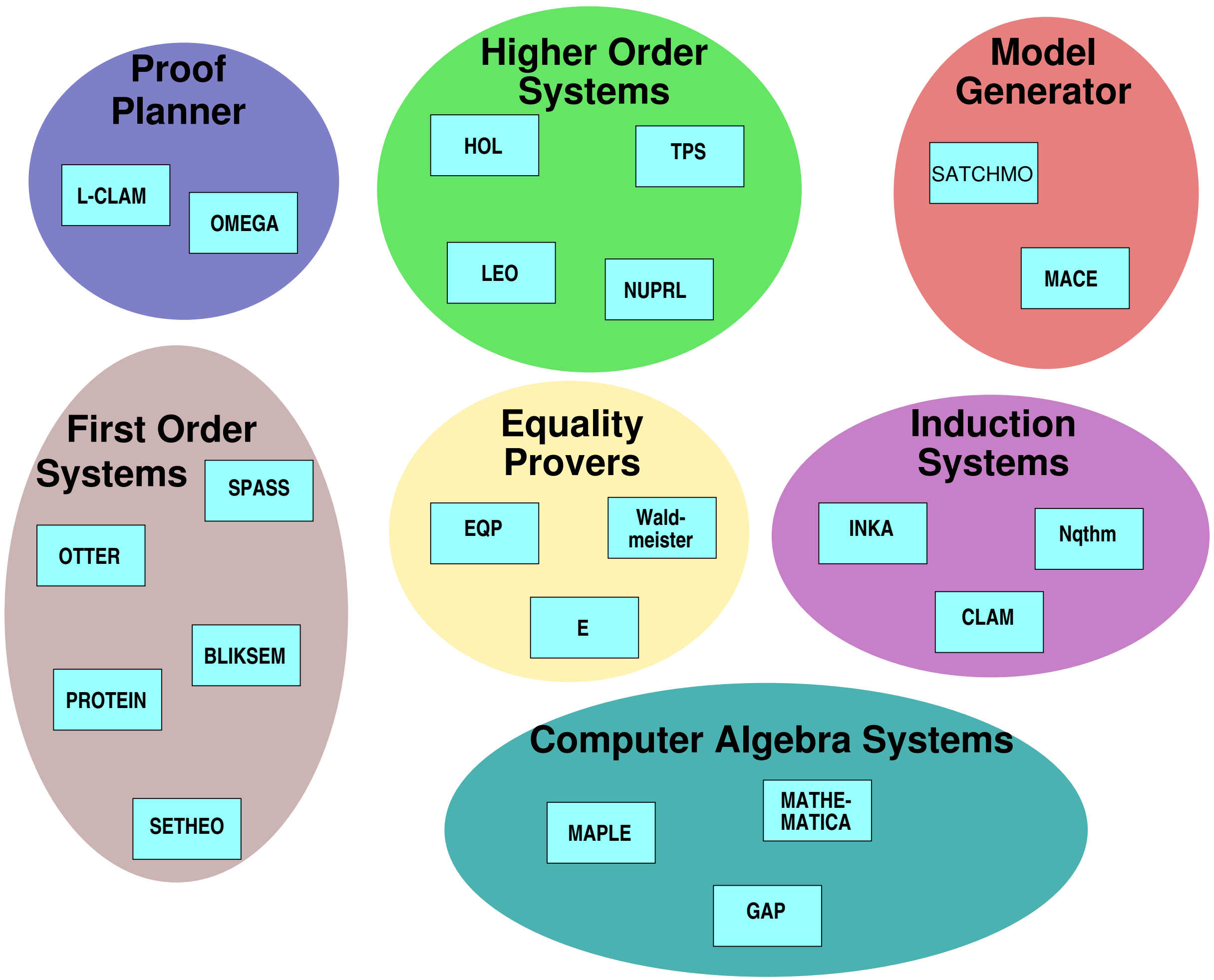
- different specialists may have to bring in their expertise and cooperate
- a communication language is required

A single mathematician

- possesses a large repertoire of specialised reasoning and problem solving techniques
- uses experience and intuition to flexibly combine them in an appropriate way

What is the best architecture for mathematical reasoning systems? **an agent based architecture?**

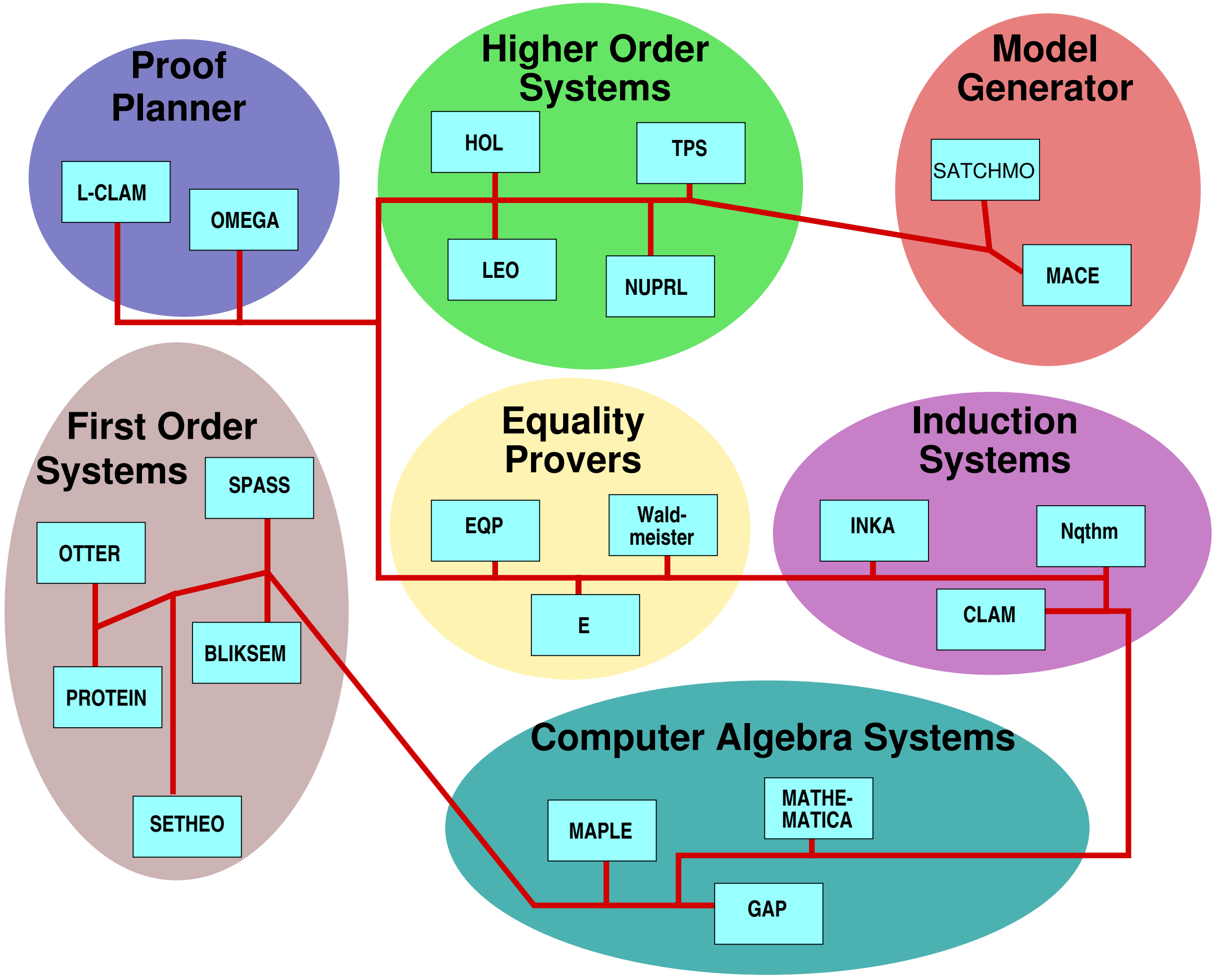
Motivation *Existing Systems*



- heterogeneous
- different niches

Motivation

Existing Systems



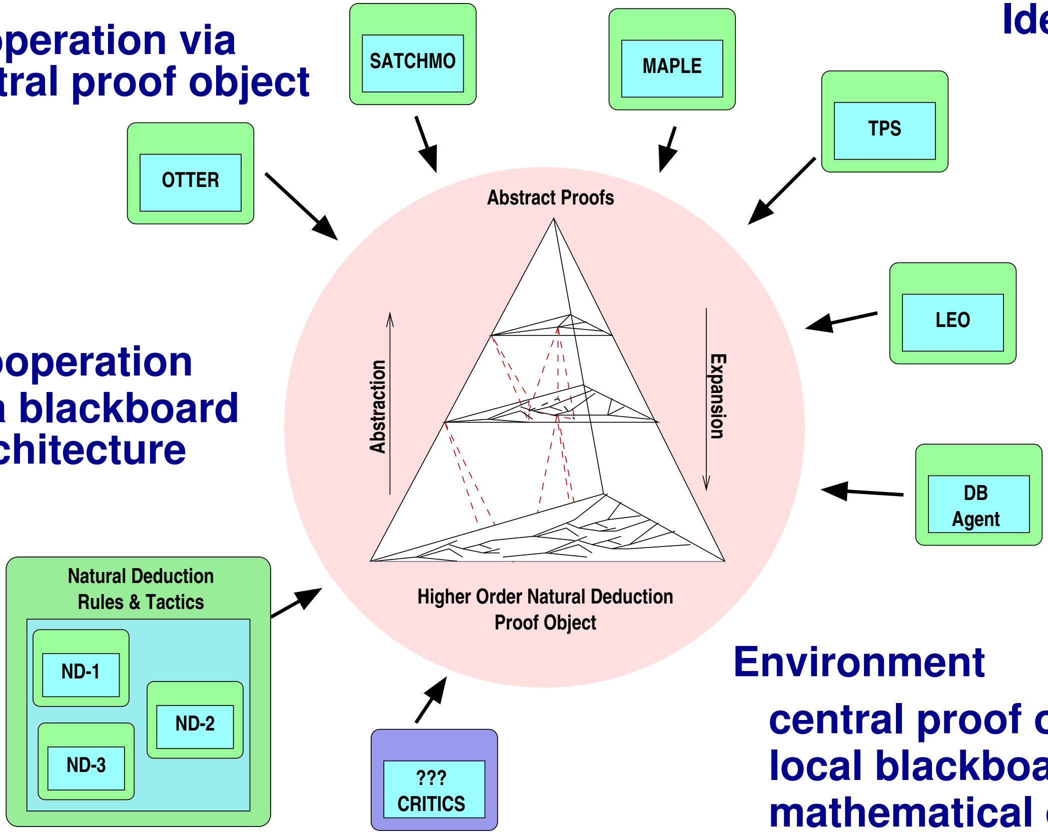
- heterogeneous
- different niches
- system networks:
  MATHWEB, PROSPER
- communication problem
- inflexible applications

How to realise a flexible interplay?

Motivation

Flexible Integration

Cooperation via central proof object



Cooperation via blackboard architecture

ND-Backbone

Idea
- **decentralised control**

Agents
- **reactive**
- **proactive**
- **heterogenous**
- **simple & complex**
- **cooperative & competetive**
- **distributed via MathWeb**
- **run-time definable**
- **resource adapted**

Environment
- **central proof object**
- **local blackboards**
- **mathematical database**

Example

HO- and FO-ATP

Higher Order ATP with LEO

C_1: favourite-numbers($\lambda x.$ odd(x) $\wedge$ square(x))

unifies (semantically) with

C_2: $\neg$favourite-numbers($\lambda x.$ square(x) $\wedge$ (square(x) $\Rightarrow$ odd(x)))

iff the following set of first order clauses can be contradicted

First Order ATP with OTTER

square(N)

$\neg$odd(N) $\vee$ $\neg$square(N)

odd(N) $\vee$ $\neg$square(N)

Example

HO- and FO-ATP

fn $=$ **favourite-numbers**

o $=$ **odd**

s $=$ **square**

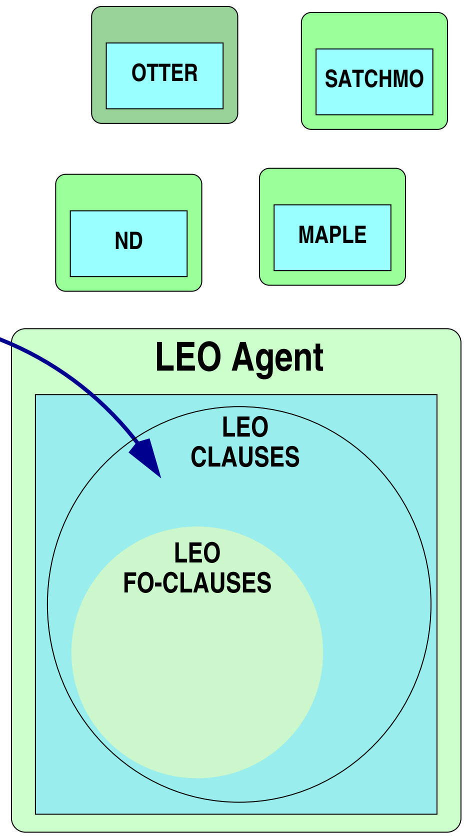
$C_1 : \mathbf{fn}(\lambda x.\, \mathbf{o}(x) \wedge \mathbf{s}(x))$

$C_2 : \neg\mathbf{fn}(\lambda x.\, \mathbf{s}(x) \wedge (\mathbf{s}(x) \Rightarrow \mathbf{o}(x)))$

$$\mathbf{fn}(\lambda x.\, o(x) \wedge \mathbf{s}(x))$$

$$\vdots\ ?$$

$$\mathbf{fn}(\lambda x.\, \mathbf{s}(x) \wedge (\mathbf{s}(x) \Rightarrow o(x)))$$



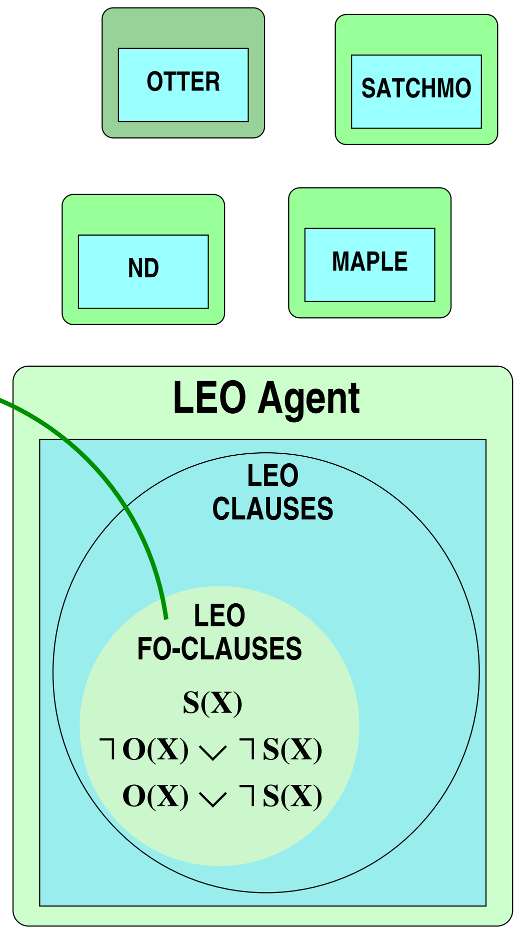
Example

HO- and FO-ATP

$$\frac{\begin{array}{c}\textsf{fn}(\lambda x.\, \textsf{o}(x) \wedge \textsf{s}(x)) \\ \neg\textsf{fn}(\lambda x.\, \textsf{s}(x) \wedge (\textsf{s}(x) \Rightarrow \textsf{o}(x)))\end{array}}{\textsf{s}(N) \wedge (\neg\textsf{o}(N) \vee \neg\textsf{s}(N)) \wedge (\textsf{o}(N) \vee \neg\textsf{s}(N))}\ \mathit{LEO}$$

$$\vdots\ ?$$

$$\frac{\bot}{\textsf{fn}(\lambda x.\, \textsf{s}(x) \wedge (\textsf{s}(x) \Rightarrow \textsf{o}(x)))}\ \mathit{indirect}$$



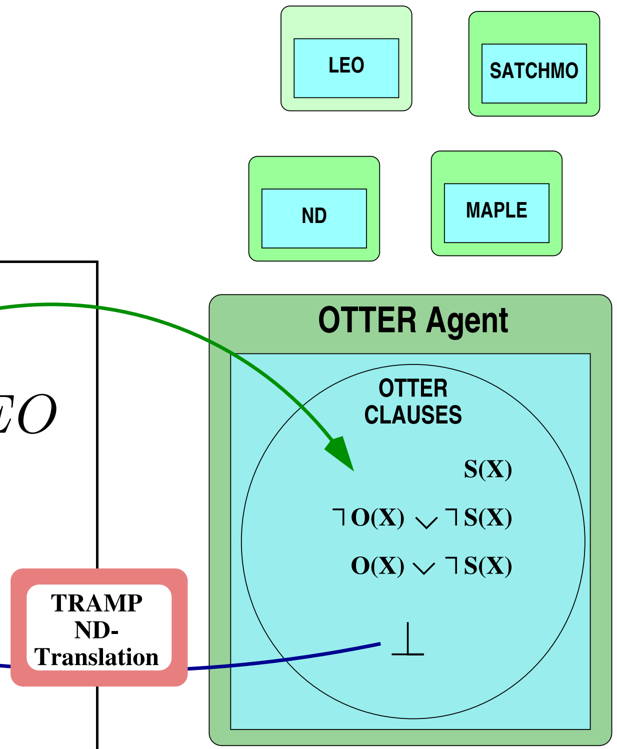
Example

HO- and FO-ATP

$$\frac{\mathbf{fn}(\lambda x.\, \mathbf{o}(x) \land \mathbf{s}(x)) \qquad \neg\mathbf{fn}(\lambda x.\, \mathbf{s}(x) \land (\mathbf{s}(x) \Rightarrow \mathbf{o}(x)))}{\mathbf{s}(N) \land (\neg\mathbf{o}(N) \lor \neg\mathbf{s}(N)) \land (\mathbf{o}(N) \lor \neg\mathbf{s}(N))}\ LEO$$

Translated OTTER Proof

$$\frac{\bot}{\mathbf{fn}(\lambda x.\, \mathbf{s}(x) \land (\mathbf{s}(x) \Rightarrow \mathbf{o}(x)))}\ indirect$$

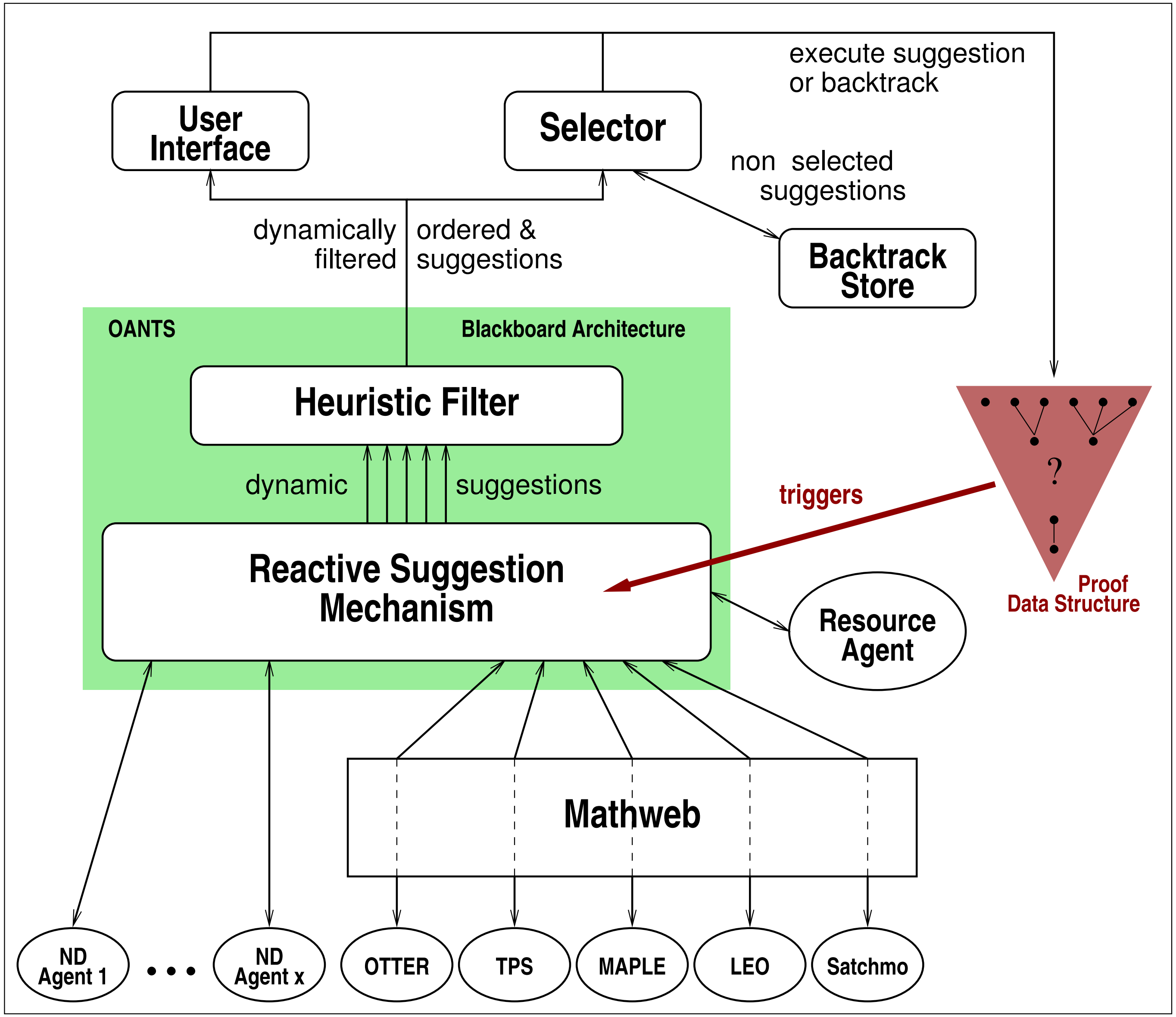


ΩANTS — *Agent based Theorem Prover*

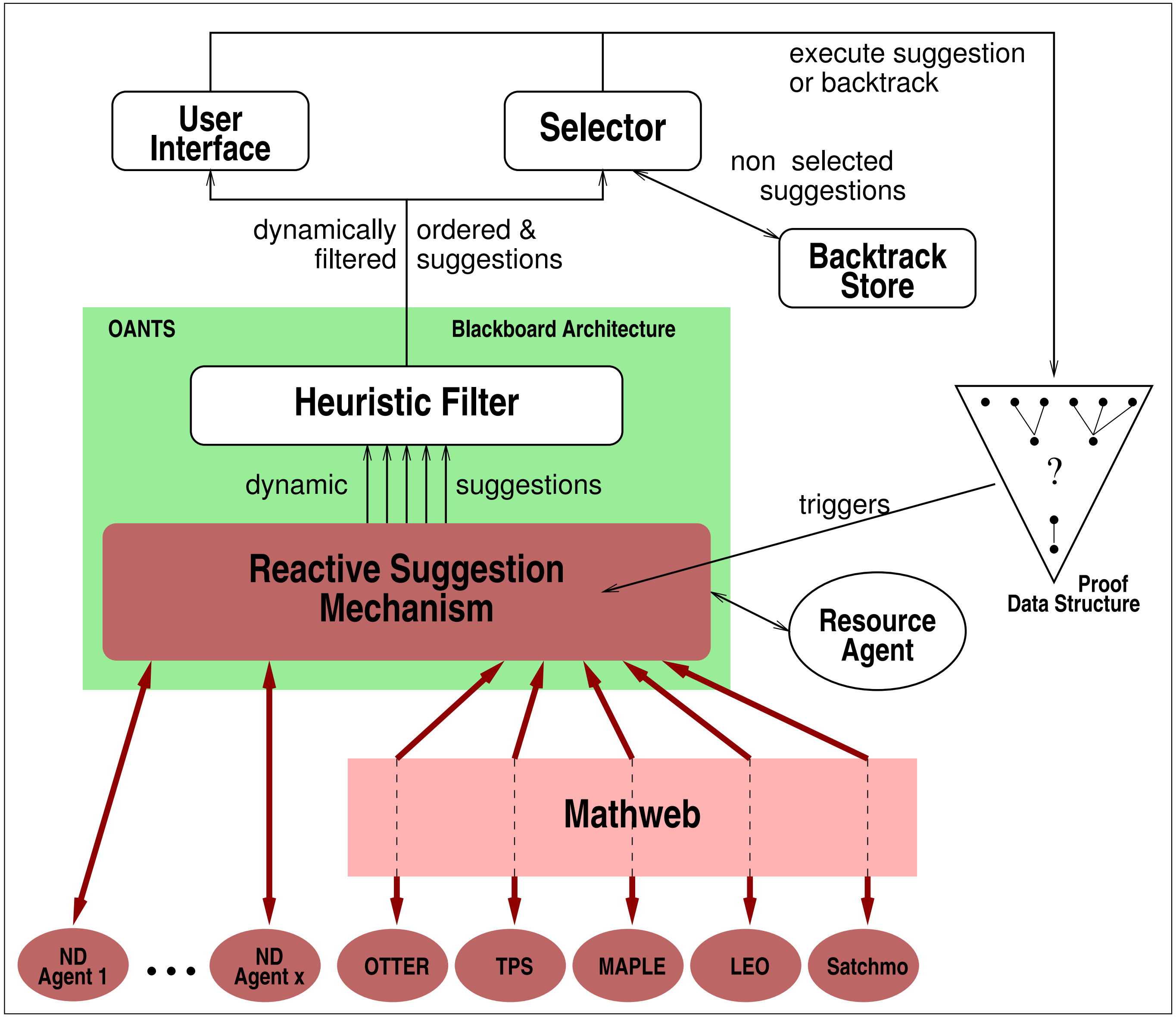
Main Components:

- ΩMEGA-System (Saarbrücken); ΩMEGA's proof data structure PDS
- ΩANTS blackboard architecture
- MATHWEB-System developed in Saarbrücken
- Various external systems integrated to ΩMEGA
- Translation modules like
  - TRAMP (FO resolution $\Longrightarrow$ ND)
  - SAPPER (CAS $\Longrightarrow$ ND)
- Calculus NIC (Carnegie Mellon University) for efficient natural deduction proof search

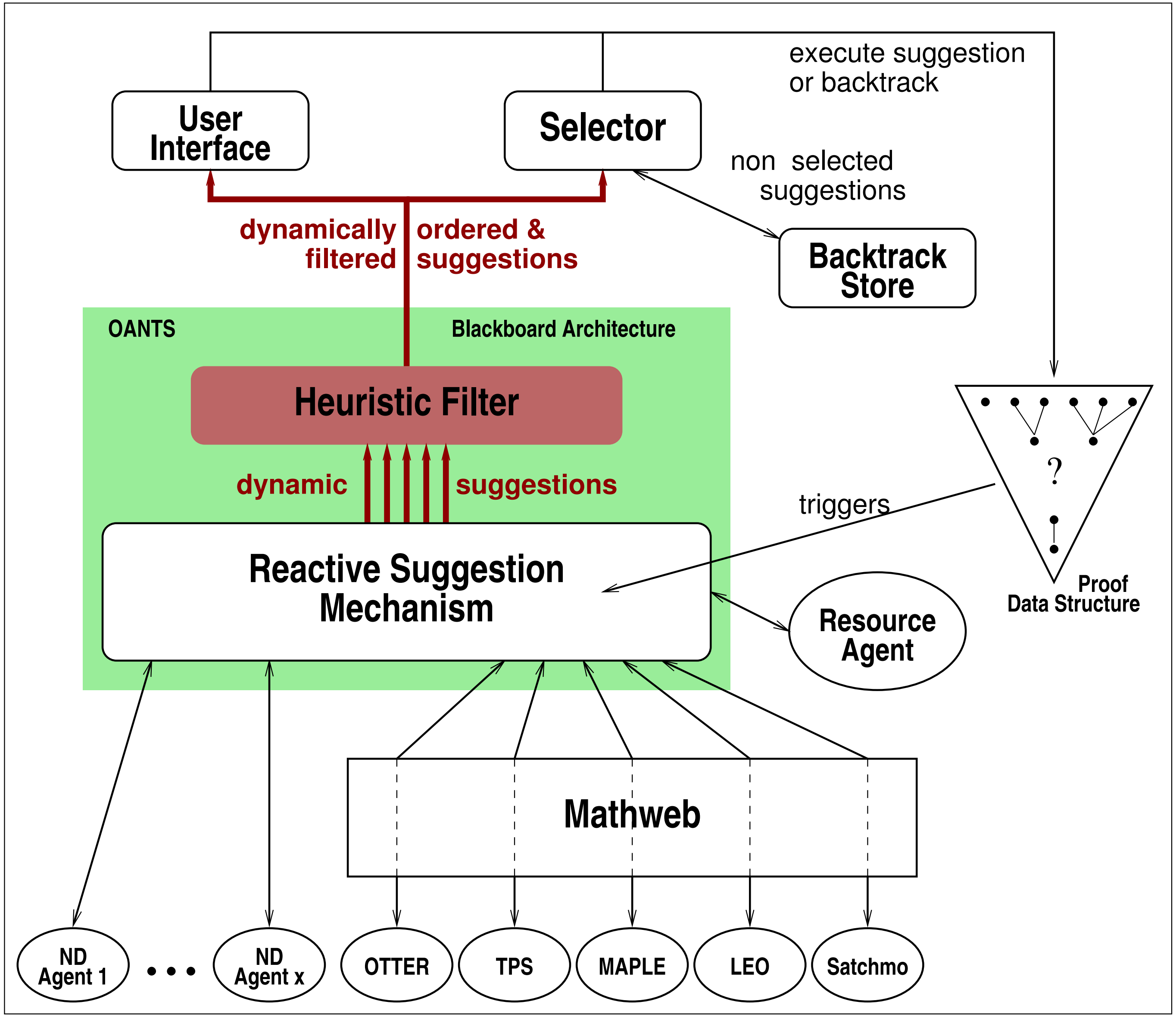
ΩANTS Agent based Theorem Prover



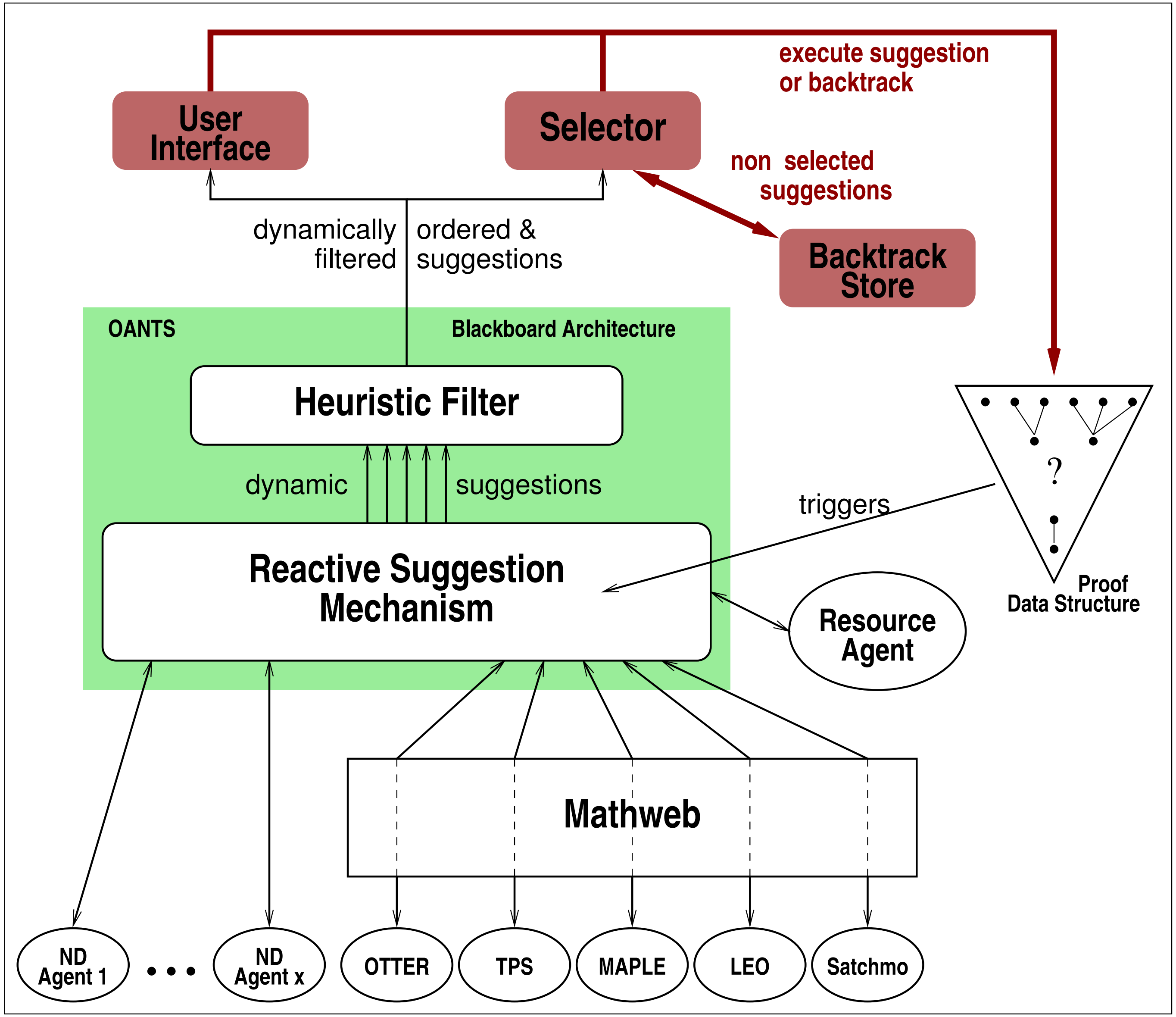
ΩANTS Agent based Theorem Prover



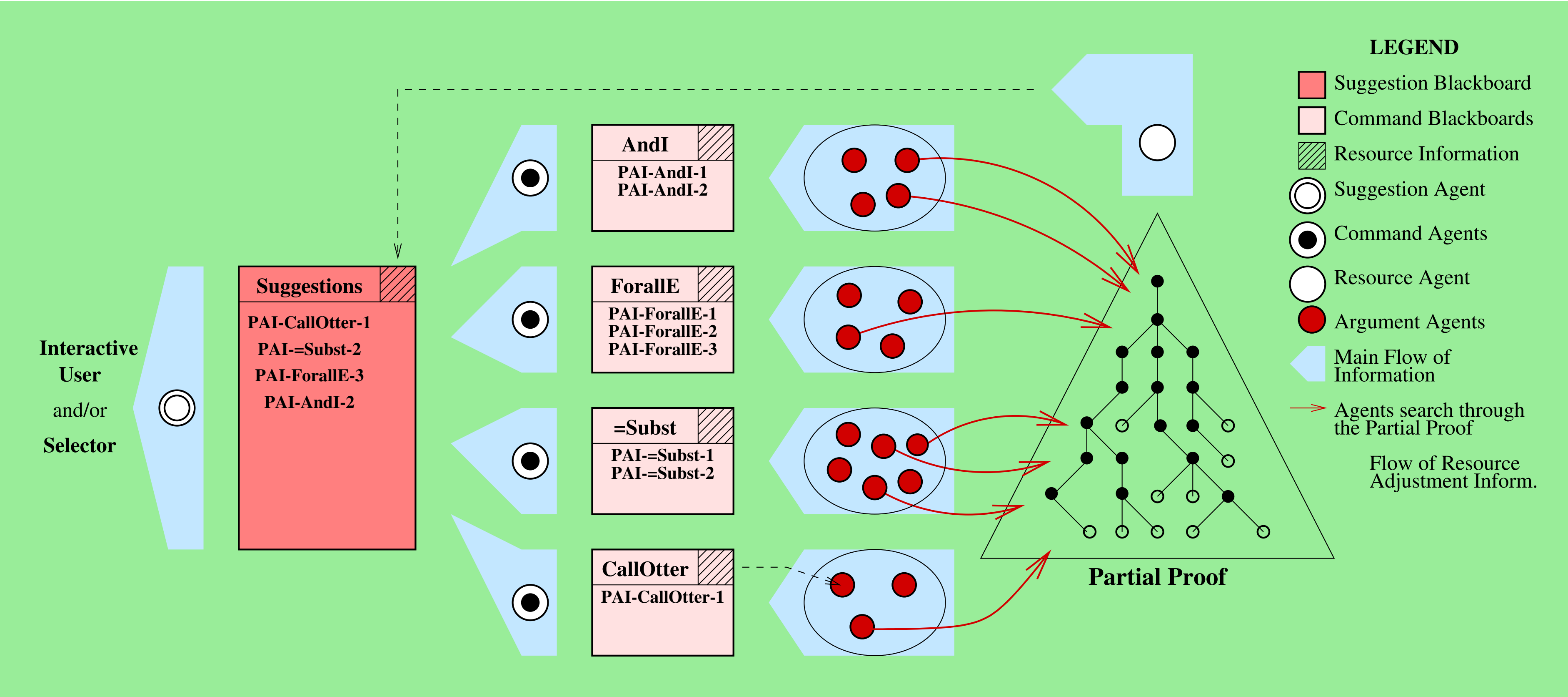
ΩANTS Theorem Prover



ΩANTS Theorem Prover



ΩANTS — Blackboard Architecture



Task: Which commands (rules, tactics, external systems) are promising in the current proof state?

$$\forall x.\,\mathsf{fn}(x)$$
$$\mathsf{fn}(N) \Rightarrow \mathsf{o}(N) \wedge \mathsf{s}(N)$$
$$\mathsf{o}(N)$$
$$\mathsf{s}(N)$$
$$\vdots\ ?$$
$$\mathsf{o}(N) \wedge \mathsf{s}(N)$$

$$\frac{\texttt{Left}\!:\!A \quad \texttt{Right}\!:\!B}{\texttt{Conj}\!:\!A \wedge B}\ \wedge E$$

$$\frac{\texttt{Ant}\!:\!A \quad \texttt{Imp}\!:\!B \Rightarrow C}{\texttt{Succ}\!:\!C}\ \texttt{mp-mod}\ (A \rightarrow B)$$

∧E-Agent-1
search for: Conj
required:
excluded: Left,Right

∧E-Agent-2
search for: Conj
required: Lef
excluded: Right

mp-mod-Agent-1
search for: Succ,Imp
required:
excluded: Ant

$$\frac{\texttt{Left}\!:\!A \quad \texttt{Right}\!:\!B}{\texttt{Conj}\!:\!\mathsf{o}(N) \wedge \mathsf{s}(N)}\ \wedge E$$

$$\frac{\texttt{Ant}\!:\!A \quad \texttt{Imp}\!:\!\mathsf{fn}(N) \Rightarrow \mathsf{o}(N) \wedge \mathsf{s}(N)}{\texttt{Succ}\!:\!\mathsf{o}(N) \wedge \mathsf{s}(N)}\ \texttt{mp-mod}\ (A \rightarrow B)$$

$$\forall x.\, \mathbf{fn}(x)$$
$$\mathbf{fn}(N) \Rightarrow \mathbf{o}(N) \wedge \mathbf{s}(N)$$
$$\mathbf{o}(N)$$
$$\mathbf{s}(N)$$
$$\vdots\ ?$$
$$\mathbf{o}(N) \wedge \mathbf{s}(N)$$

$$\frac{\texttt{Left}\!:\!A \quad \texttt{Right}\!:\!B}{\texttt{Conj}\!:\!\mathbf{o}(N) \wedge \mathbf{s}(N)} \wedge E$$

$$\frac{\texttt{Ant}\!:\!A \quad \texttt{Imp}\!:\!\mathbf{fn}(N) \Rightarrow \mathbf{o}(N) \wedge \mathbf{s}(N)}{\texttt{Succ}\!:\!\mathbf{o}(N) \wedge \mathbf{s}(N)} \text{ mp-mod } (A \rightarrow B)$$

∧E-Agent-2
search for: Left
required: Conj
excluded:

∧E-Agent-3
search for: Right
required: Conj
excluded:

mp-mod-Agent-2
search for: Ant
required: Succ
excluded:

$$\frac{\texttt{Left}\!:\!\mathbf{o}(N) \quad \texttt{Right}\!:\!\mathbf{s}(N)}{\texttt{Conj}\!:\!\mathbf{o}(N) \wedge \mathbf{s}(N)} \wedge E$$

$$\frac{\texttt{Ant}\!:\!\forall x.\, \mathbf{fn}(x) \quad \texttt{Imp}\!:\!\mathbf{fn}(N) \Rightarrow \mathbf{o}(N) \wedge \mathbf{s}(N)}{\texttt{Succ}\!:\!\mathbf{o}(N) \wedge \mathbf{s}(N)} \text{ mp-mod } (A \rightarrow B)$$

ΩANTS — *Various Further Aspects*

- **Declarative agent specification language**

  uniform way to define new agents
  run-time modifiability of agent societies

- **Attempt to a formal semantics**

  mapping agent declarations to simply typed λ-calculus
  some properties of agents and agent societies can be modelled

- **Self-evaluation of agents**

  knowledge about their own performance
  this knowledge is broadcasted via the blackboards
  explicit resource reasoning on informed layer

ΩANTS Resource Adapted Behaviour

Selection clock speed:

determines resource computation time ct for the agents

- ct high — automatic proof by single ATP
- ct medium — cooperative proof
- ct low — (unsuccessful) attack at ND level

First experiments (resource adaptivity in interactive sessions)

agents decide to get inactive/active wrt varying clock speed

Results

Example Classes

Ex1 **higher order ATP and first order ATP**

$\forall x, y, z. (x = y \cup z) \Leftrightarrow (y \subseteq x \wedge z \subseteq x \wedge \forall v. (y \subseteq v \wedge z \subseteq v) \Rightarrow (x \subseteq v))$

Ex2 **ND based TP, propositional ATP, and model generation**

$\forall x. \forall y. \forall z. ((x \cup y) \cap z) = (x \cap z) \cup (y \cap z)$ **10000 Examples**

$\forall x. \forall y. \forall z. ((x \cup y) \cup z) = (x \cap z) \cup (y \cap z)$ **988 valid / 9012 invalid**

Ex3 **computer algebra systems and higher order ATP**

$\{x | x > gcd(10,8) \wedge x < lcm(10,8)\} = \{x | x < 40\} \cap \{x | x > 2\}$

Ex4 **ND and tactical based TP, first-order ATP**

$\ldots$ *group-definition-1* $\ldots \Leftrightarrow \ldots$ *group-definition-2* $\ldots$

Results

Ex2: ND, PL-ATP, model generation

Conc.	$\vdash \forall x.\forall y.\forall z.((x \cup y) \cap z) = (x \cap z) \cup (y \cap z)$	(**Forall-I L1**)
	$\ldots$	
L3.	$\vdash ((X \cup Y) \cap Z) = (X \cap Z) \cup (Y \cap Z)$	(**Set-Ext L4**)
L4.	$\vdash \forall e.e \in ((X \cup Y) \cap Z) \leftrightarrow e \in (X \cap Z) \cup (Y \cap Z)$	(**Forall-I L5**)
L5.	$\vdash E \in ((X \cup Y) \cap Z) \leftrightarrow E \in (X \cap Z) \cup (Y \cap Z)$	(**Def L6**)
	$\ldots$	
L8.	$\vdash ((E \in X \vee E \in Y) \wedge E \in Z) \leftrightarrow ((E \in X \wedge E \in Z) \vee (E \in Y \wedge E \in Z))$	(**OTTER**)

Theorem

Results

Ex2: ND, PL-ATP, model generation

Conc.	$\vdash \forall x. \forall y. \forall z. ((x \cup y) \cup z) = (x \cap z) \cup (y \cap z)$	(**Forall-I L1**)
	$\dots$	
L3.	$\vdash ((X \cup Y) \cup Z) = (X \cap Z) \cup (Y \cap Z)$	(**Set-Ext L4**)
L4.	$\vdash \forall e. e \in ((X \cup Y) \cup Z) \leftrightarrow e \in (X \cap Z) \cup (Y \cap Z)$	(**Forall-I L5**)
L5.	$\vdash E \in ((X \cup Y) \cup Z) \leftrightarrow E \in (X \cap Z) \cup (Y \cap Z)$	(**Def L6**)
	$\dots$	
L8.	$\vdash ((E \in X \vee E \in Y) \vee E \in Z) \leftrightarrow ((E \in X \wedge E \in Z) \vee (E \in Y \wedge E \in Z))$	(**SATCHMO**)

Counter Model: $G \in Z \wedge G \notin X \wedge G \notin Y$

Results

Ex3: ND based TP, CAS and HO-ATP

$$\{x|x > gcd(10,8) \wedge x < lcm(10,8)\} = \{x|x < 40\} \cap \{x|x > 2\}$$

Conc.	$\vdash (\lambda x_\blacksquare x > gcd(10,8) \wedge x < lcm(10,8)) = (\lambda x_\blacksquare x < 40) \cap (\lambda x_\blacksquare x > 2)$	(**CAS L1**)
L1.	$\vdash (\lambda x_\blacksquare x > 2 \wedge x < 40) = (\lambda x_\blacksquare x < 40) \cap (\lambda x_\blacksquare x > 2)$	(**Def L3**)
L3.	$\vdash (\lambda x_\blacksquare x > 2 \wedge x < 40) = (\lambda x_\blacksquare x < 40 \wedge x > 2)$	(**LEO**)

Related Work

- **Overview of parallel & distributed theorem proving [Bonacina 2000]**
- TECHS **approach [Denzinger and Fuchs 1999]**
  heterogeneous first-order systems, filtered exchange of clauses
  no higher-order systems and no CAS
  no explicit proof object
  no user orientation
- **Open approach to concurrent theorem proving [Fisher 1997]**
- **Multi agent proof-planning [Fisher and Ireland 1998]**
- **Agent planning architectures, e.g. [Wilkins and Myers 1998]**
- . . . **agent based architectures** . . .

Conclusion

Agent based architecture

application to mathematical reasoning (HO)
with heterogeneous external systems
flexible integration of new agents
cooperation & competition
supports automation and interaction
abstract inferences & low level ND inferences
resource adapted & adaptive

Typical applications

domains where different specialist systems are required
higher-order examples with first-oder subtasks
exploration of new domains

Architecture not restricted to theorem proving

central proof object $\longrightarrow$ knowledge base
proof rules, tactics, external systems $\longrightarrow$ production rules

Problems and Future Work

Long-term goals

solving the communication problem

choice of interlingua between agents

adding or-parallelism

full integration with proof planning

critical (reflecting) agents

dynamic clustering of agents

experiment: iterated learning of tactics/methods

Short-term goals

employ counterexample information for early backtracking

counterexamples by Venn diagrams

various technical problems (copying of PDS)