

System Description: The Metis Proof Tactic

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The Metis proof tactic for the HOL4 theorem prover [1] proves higher order logic goals using a first order proof calculus. It is implemented in Standard ML, supporting a tight integration with the rest of the HOL4 theorem prover, and is required to respond within a few seconds to be useful during interactive proof.

The steps of its operation are as follows:

1. The initial higher order logic goal is negated and converted by proof to conjunctive normal form. Definitional conjunctive normal form is used to avoid exponential blow-up (occasionally encountered in practice).
2. A suitable logical interface (m, t) is selected, consisting of a mapping m from higher order logic formulas to first order clauses, and also a translation t that lifts first order refutations to higher order logic proofs [2]. The conjuncts are mapped to first order logic clauses.
3. A refutation of the clauses is found, where the search is performed using the ordered paramodulation calculus [3].
4. The first order refutation is lifted to a higher order logic proof of the normalized goal, completing the proof of the initial goal.

The Metis proof tactic is effective on many classes of higher order logic goal, particularly those that require a combination of deductive and equality reasoning. It automatically selects an interface for the goal, first trying a fast one that discards type information, and then a more robust one that includes type information in the first order clauses. A syntactic check detects whether the goal contains higher order features such as quantification over functions, and if so an interface is selected that maps higher order logic function application to a first order function symbol (i.e., $f(x)$ is mapped to `app(f, x)`). This automatic interface selection makes Metis more efficient and also gives it a wider coverage than might be expected of a first order proof tactic. As an indication of its popularity, the string `METIS_TAC` occurs 1,822 times in the 243,636 lines of the HOL4 sources, and work is ongoing to port it to other higher order logic theorem provers.

References

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* Supported by a Junior Research Fellowship at Magdalen College, Oxford.

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3. R. Nieuwenhuis and A. Rubio. Paramodulation-based theorem proving. In A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*, volume I, chapter 7, pages 371–443. Elsevier Science, 2001.