





SET171+3  $\forall X_0 ; Y_0 ; Z_0 : X (Y Z) = (X Y) (X Z)$   
SET611+3  $\forall X_0 ; Y_0 : (X$



<b>Assumptions:</b>	$\forall B; C; x. [x \in (B \cup C) \rightarrow x \in B \vee x \in C]$	(1)
	$\forall B; C; x. [x \in (B \cap C) \rightarrow x \in B \wedge x \in C]$	(2)
	$\forall B; C [B = C \rightarrow B \subseteq C \wedge C \subseteq B]$	(3)
	$\forall B; C [B \subseteq C \rightarrow C \subseteq B]$	(4)
	$\forall B; C [B \subseteq C \rightarrow C \subseteq B]$	(5)
	$\forall B; C [B \subseteq C \rightarrow \forall x. x \in B \rightarrow x \in C]$	(6)
	$\forall B; C [B = C \rightarrow \forall x. x \in B \rightarrow x \in C]$	(7)
<b>Proof Goal:</b>	$\forall B; C; D [B \subseteq (C \cap D) \rightarrow (B \subseteq C) \wedge (B \subseteq D)]$	(8)

Table 3. SET171+3: The First-Order TPTP En/BPC 1 ID EI 04)

(1) $\forall B; C; D: B \rightarrow (C \rightarrow D) = (B \rightarrow C) \rightarrow (B \rightarrow D)$	clause initialization def.-expansion, cnf B; C; D Skolem const.
(2) $[(\exists x: Bx \rightarrow (Cx \rightarrow Dx)) = (\exists x: (Bx \rightarrow Cx) \rightarrow (Cx \rightarrow Dx))]^F$	unification constraint
(3) $[(\exists x: Bx \rightarrow (Cx \rightarrow Dx)) =^? (\exists x: (Bx \rightarrow Cx) \rightarrow (Cx \rightarrow Dx))]$	f-extensionality x new Skolem constant
(4) $[(Bx \rightarrow (Cx \rightarrow Dx)) =^? ((Bx \rightarrow Cx) \rightarrow (Cx \rightarrow Dx))]$	B-extensionality
(5) $[(Bx \rightarrow (Cx \rightarrow Dx)) \rightarrow ((Bx \rightarrow Cx) \rightarrow (Cx \rightarrow Dx))]^F$	cnf, factor., subsumption
(6) $[Bx]^F$	
(7) $[Bx]^T \ [Cx]^T$	propositional problem!
(8) $[Bx]^T \ [Dx]^T$	
(9) $[Cx]^F \ [Dx]^F$	propositional reasoning
(10)	

Table 4. Problem SET171+3: Solution in LEO

For the experiments with Leo and the cooperation of Leo with the first-order theorem prover Bliskem,  $\lambda$ -abstraction as well as the extensionality treatment inherent in Leo's calculus [4] is used. This enables a theorem prover Henkin-complete proof system for set theory. In the above example SET171+3, Leo generates the application of functional extensionality to push unification constraints down to base terms.



Some problems are immediately mapped by recursive definition expansion (without extensionality reasoning) and normalisation into the empty clause such that proof search does not even start; an example is SET646+3. Some problems require several rounds of extensionality processing within Leo's set-of-support based proof search procedure before the bag of first-order like clauses turns into a refutable

ing systems such as automated theorem provers, computer algebra systems, or model generators into the architecture.

### 3.1 Cooperation via multiple inference rules

The use of the external systems is modelled by inference

### **3.2 Cooperation via a single inference rule**

**In order to overcome the problem**

layer of the ants blackboard architecture. This is not an ad hoc solution, but

{ When the above suggestion of a successful joint proof attempt is



