

Title: Automated Reasoning in Fragments of Church's Type Theory

Abstract: Theorem Proving in Church's Type Theory can be factored into several dimensions including unification, set instantiations, extensionality, infinity, description and choice. Early efforts to automate proof search worked in a fragment of Church's Type Theory called elementary type theory. Elementary type theory combines simply-typed lambda-calculus with logical constants, but does not include extensionality, infinity, description or choice. The techniques for search in elementary type theory combine first-order methods with higher-order pre-unification and an enumeration technique (primitive substitutions) for handling set substitutions. The use of primitive substitutions to enumerate set substitutions is not a practical technique, but does lead to a complete procedure (in principle). More recent efforts have incorporated extensionality into search. To some degree, one can restrict primitive substitutions when searching for extensional set instantiations. However, the need to perform primitive substitutions remains the most impractical part of the procedure. In practice, implementations usually limit which primitive substitutions are generated. Theoretically, this means the procedure is searching in a fragment of Church's Type Theory with missing logical constants. Such fragments have a well-defined model theory. Consequently, one can consider precisely which theorems of Church's Type Theory fail to be theorems in various fragments.